

[This question paper contains 4 printed pages.]

Your Roll No.....

22/5/25 R

Sr. No. of Question Paper : 1345

J

Unique Paper Code : 32351602

Name of the Paper : Ring Theory and Linear Algebra-II

Name of the Course : B.Sc. (H) Mathematics
(CBCS-LOCF)

Semester : VI

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the questions are compulsory.
3. Attempt any two parts from each question.
4. Marks of each part are indicated.

1. (a) Let F be a field. Show $F[x]$ is a principal ideal domain.

(b) (i) Is $x^3 + 1$ irreducible over $\mathbb{Z}_7$.

(ii) Show $x^4 + 1$ is reducible over $\mathbb{R}$.

(c) State and Prove Gauss Lemma.

(6,3×2,6)

P.T.O.

2. (a) Show $\frac{\mathbb{R}[x]}{\langle x^2+1 \rangle} \cong \mathbb{C}$. Hence, Prove $\langle x^2+1 \rangle$ is maximal in $\mathbb{R}[x]$.

(b) Is every Unique Factorization Domain a Principal Ideal Domain? Prove or give a Counterexample.

(c) Show $\mathbb{Z}[\sqrt{-5}]$ is not a Unique Factorization Domain. (Consider the factorization of the element 6). (6.5+6.5+6.5)

3. (a) Let $V = \mathbb{R}^3$ and define $f_1, f_2, f_3 \in V^*$ as follows

$$f_1(x, y, z) = x - 2y, \quad f_2(x, y, z) = x + y + z \text{ and } f_3(x, y, z) = y - 3z.$$

Prove that $\{f_1, f_2, f_3\}$ is a basis of V^* and then find a basis for V for which it is the dual basis.

(b) Show that the matrix $A = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$ is diagonalizable

and hence find an invertible matrix Q and a diagonal matrix D such that $Q^{-1}AQ = D$.

(c) Test the linear operator $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$, $T(a, b, c) = (4a + c, 2a + 3b + 2c, a + 4c)$ for diagonalizability and if it is diagonalizable, find a basis β for V such that $[T]_\beta$ is a diagonal matrix. (6,6,6)

4. (a) For the linear operator $T: \mathbb{R}^4 \rightarrow \mathbb{R}^4$, $T(a, b, c, d) = (a + b, b - c, a + c, a + d)$, determine the T - cyclic subspace W of $\mathbb{R}^4$ generated by $e_1 = (1, 0, 0, 0)$. Also, find an ordered basis of W and characteristic polynomial of the operator T_W .

- (b) State Cayley-Hamilton theory and verify it for the linear operator

$$T: \mathbb{R}^3 \rightarrow \mathbb{R}^3, T(a, b, c) = (a + b, b + c, b - c).$$

- (c) Let $V = P_1(\mathbb{R})$ and $W = \mathbb{R}^2$ with respective standard bases β and γ . Define $T: V \rightarrow W$ by $T(p(x)) = (p(0) - 2p(1), p(0) + p'(0))$, where $p'(x)$ is the derivative of $p(x)$.

- (i) For $f(a, b) = a - 2b$, $f \in W^*$, compute $T^t(f)$.

- (ii) Compute $\left[T^t \right]_{\gamma^*}^{\beta^*}$ by finding scalars a, b, c and d such that $T^t(f_1) = af_3 + bf_4$ and $T^t(f_2) = cf_3 + df_4$, where $\beta^* = \{f_3, f_4\}$ and $\gamma^* = \{f_1, f_2\}$ are the respective dual bases.

(6.5, 6.5, 3+3.5)

5. (a) Show that in a complex inner product space V over a field F . If $x, y \in V$, then following identities hold :

$$(i) \langle x, y \rangle = \frac{1}{4} \|x + y\|^2 - \frac{1}{4} \|x - y\|^2 \text{ if } F = \mathbb{R}$$

$$(ii) \langle x, y \rangle = \frac{1}{4} \sum_{k=1}^4 \|x + i^k y\|^2 \text{ if } F = \mathbb{C}, \text{ where } i^2 = -1.$$

P.T.O.

- (b) Let V be an inner product space, and let $S = \{v_1, v_2, v_3, \dots, v_n\}$ be an orthonormal subset of V . Prove the Bessel's Inequality :

$$\|x\|^2 \geq \sum_{i=1}^n |\langle x, v_i \rangle|^2 \quad \text{for any } x \in V.$$

- (c) Prove that an orthogonal subset of a finite dimensional inner product space V can be extended to an orthonormal basis for V . Hence, or otherwise prove that for any subspace W of a finite dimensional inner product space V , $\dim(V) = \dim(W) + \dim(W^\perp)$. (3+3,6,6)

6. (a) Let T be a linear operator on a complex finite dimensional inner product space V with an adjoint T^* . Prove that

(i) T is self-adjoint if and only if $\langle Tx, x \rangle$ is real for all $x \in V$.

(ii) If $\langle Tx, x \rangle = 0$ for all $x \in V$, then $T = T_0$, the zero operator on V .

- (b) Let $V = M_{2 \times 2}(\mathbb{R})$ and $T: V \rightarrow V$ be a linear operator given by $T(A) = A^T$. Determine whether T is normal, self-adjoint, or neither. If possible, produce an orthogonal basis of eigenvectors of T for V and list the corresponding eigenvalues.

- (c) Let U be a Unitary operator on an inner product space V and let W be a finite dimensional U -invariant subspace of V . Then, prove that

(i) $U(W) = W$

(ii) $W^\perp$ is U -invariant. (3+3.5,6.5,3+3.5)

[This question paper contains 8 printed pages.]



Your Roll No.....

1015 May
2025

Sr. No. of Question Paper : 1353

J

Unique Paper Code : 32351601

Name of the Paper : Complex Analysis

Name of the Course : **B.Sc. (H) Mathematics**
CBCS-LOCF

Semester : VI – Core

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. Use of Calculator is not allowed.

P.T.O.

1. (a) Find and sketch the image of each branch of the hyperbola $x^2 - y^2 = c$, $c > 0$ under the transformation $w = z^2$. (6)

(b) Prove that (i) $\lim_{z \rightarrow 0} \frac{z^{-2}}{z} = 0$ (ii) $\lim_{z \rightarrow \infty} \frac{z^2 + 1}{z - 1} = \infty$ (3+3)

- (c) Prove that if $f'(z) = 0$ everywhere in a domain D , then $f(z)$ must be constant throughout D . (6)

- (d) Show that the function $f(z) = \cosh x \cos y + i \sinh x \sin y$ is entire. (6)

2. (a) (i) Find all values of z such that $e^z = -2$.

(ii) Show that if e^z is real, then $\text{Im } z = n\pi$, $n \in \mathbb{Z}$. (3+3.5)

- (b) Show that $\sin z = 0$ if and only if $z = n\pi$, $n \in \mathbb{Z}$. (6.5)

(c) Show that

$$(i) \operatorname{Log} [(1+i)^2] = 2 \operatorname{Log} (1+i)$$

$$(ii) \operatorname{Log} [(-1+i)^2] \neq 2 \operatorname{Log} (-1+i).$$

(3+3.5)

(d) Show that

$$(i) \log e = 1 + 2n\pi i, \quad n \in \mathbb{Z}.$$

$$(ii) \log i = \left(2n + \frac{1}{2}\right)\pi i, \quad n \in \mathbb{Z}. \quad (3+3.5)$$

3. (a) Let $w(t) = u(t) + i v(t)$ be a complex-valued function of a real parameter t .

(i) Define the definite integral of $w(t)$ over $a \leq t \leq b$ and evaluate

$$\int_1^2 \left(\frac{1}{t} - i\right)^2 dt.$$

- (ii) Show that if $w(t)$ denotes a continuous function on $[-a, a]$ which is even; that is, $w(-t) = w(t)$ for each point t in $[-a, a]$, then

$$\int_{-a}^a w(t) dt = 2 \int_0^a w(t) dt. \quad (3+3)$$

- (b) What do you mean by differentiable arc and its length? Show that the length of an arc C remain invariant under change in the representation for C . (2+4=6)

- (c) Let $f(z) = \pi \exp(\pi \bar{z})$ and C be the boundary of the square with vertices at the points $0, 1, 1 + i$ and i , the orientation of C being the counter clockwise direction. Using parametric representation of C ,

evaluate $\int_C f(z) dz$. (6)

(d) Without evaluating the integral, show that

$$\left| \int_C \frac{z+4}{z^3-1} dz \right| \leq \frac{6\pi}{7}$$

where C is the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$ that lies in the first quadrant.

(6)

4. (a) Use the method of antiderivatives to show that for every contour C extending from a point z_1 to z_2 ,

$$\int_C z^n dz = \frac{z_2^{n+1} - z_1^{n+1}}{n+1} \quad \text{for each } n = 0, 1, 2, \dots$$

State the corresponding result used.

(4.5+2=6.5)

- (b) State Cauchy Goursat's theorem and hence evaluate

(i) $\int_C \text{Log}(z+2) dz$

P.T.O.

$$(ii) \int_C \frac{z^2}{z+3} dz$$

where C is the unit circle $|z| = 1$ in either direction.

(1.5+2.5+2.5=6.5)

(c) Show that if f is analytic at a given point then its derivatives of all higher orders are analytic there too. (6.5)

(d) State and prove fundamental theorem of algebra. (6.5)

5. (a) Suppose that the sequence (z_n) , where $z_n = x_n + iy_n$, $n = 1, 2, \dots$ and $z = x + iy$ then prove that $\lim_{n \rightarrow \infty} z_n = z \Leftrightarrow \lim_{n \rightarrow \infty} x_n = x$ and $\lim_{n \rightarrow \infty} y_n = y$, and hence check the convergence behavior of complex

sequence $z_n = \frac{2}{n^3} + 5i$, ($n = 1, 2, \dots$).

(3.5+3=6.5)

(b) Find Maclaurin series representation of $f(z) = \cosh z$. (6.5)

- (c) If a series $\sum_{n=0}^{\infty} a_n (z - z_0)^n$ converges to $f(z)$ at all points interior to some circle $|z - z_0| = R$, then prove that it is the Taylor series for the function $f(z)$ in powers of $z - z_0$. (6.5)

- (d) Prove that when

$$z \neq 0, \frac{\sin(z^2)}{z^4} = \frac{1}{z^2} - \frac{z^2}{3!} + \frac{z^6}{5!} - \frac{z^{10}}{7!} + \dots \quad (6.5)$$

6. (a) Prove that if a sequence of complex numbers converges, then its n^{th} term converges to zero as $n \rightarrow \infty$. is the converse true? Justify. (6)

- (b) Derive the Laurent series expansion for the

function $f(z) = \frac{1}{e^z}$ and hence show that

$\int_{\Gamma} \frac{1}{e^z} dz = 2\pi i$, where Γ is any positively oriented simple closed contour around the origin.

(3+3=6)

- (c) Determine the residue of the below given complex functions at $z = 0$

(i)
$$\frac{\sin hz}{z^4(1-z^2)}$$

(ii)
$$\frac{\cot z}{z^4} \quad (3+3=6)$$

- (d) Define singular point and isolated singular point of a complex function. Consider the function

$$f(z) = \frac{\sin z}{(z-1)^3(z+2)}$$

Identify all isolated singularities of the function $f(z)$. (3+3=6)

[This question paper contains 4 printed pages.]

627/5/257
Your Roll No.

Sr. No. of Question Paper : 1357

Unique Paper Code : 32357610

Name of the Paper : DSE-4 (Number Theory)

Name of the Course : CBCS (LOCF) – B.Sc. (H)
Mathematics

Semester : VI

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Attempt any three parts from each question.
4. Each part carries 5 marks.

P.T.O.

1. (a) Determine all solutions in integers of the Diophantine equation

$$54x + 21y = 906.$$

- (b) A farmer purchased 100 head of livestock for a total cost of \$4000. Prices were as follows: Calves \$120 each; lambs \$50 each; piglets \$25 each. If the farmer obtained at least one animal of each type, how many of each did he buy?
- (c) What is the remainder when the sum $1^5 + 2^5 + 3^5 + \dots + 100^5$ is divided by 4?
- (d) Solve the linear congruence $6x \equiv 15 \pmod{21}$.
2. (a) Solve the following set of simultaneous congruences

$$x \equiv 1 \pmod{3}$$

$$x \equiv 2 \pmod{5}$$

$$x \equiv 3 \pmod{7}$$

- (b) Find the solutions to the system of congruences

$$7x + 3y \equiv 10 \pmod{16}$$

$$2x + 5y \equiv 9 \pmod{16}$$

- (c) State and prove Fermat's Theorem.

(d) Prove that for any odd prime p , $1^2, 3^2, 5^2, \dots, (p-2)^2$

$$\equiv (-1)^{\frac{p+1}{2}} \pmod{p}.$$

3. (a) For any integer n , show that $\mu(n)\mu(n+1)\mu(n+2)\mu(n+3) = 0$.

(b) Let F and f be two number-theoretic functions related by the formula :

$$F(n) = \sum_{d|n} f(d)$$

Show that

$$f(n) = \sum_{d|n} \mu(d) F\left(\frac{n}{d}\right) = \sum_{d|n} \mu\left(\frac{n}{d}\right) F(d)$$

(c) Find the number of zeros with which the decimal representation of $50!$ terminates.

(d) Show that $\phi(3n) = 3\phi(n)$ if and only if $3|n$.

4. (a) For a square free integer $n > 1$, show that $\tau(n^2) = n$ if and only if $n = 3$.

(b) Define primitive root of an integer $n > 1$. Find all the primitive roots of 9.

- (c) If p is a prime, show that the product of the $\phi(p-1)$ primitive roots of p is congruent modulo p to $(-1)^{\phi(p-1)}$.
- (d) Find a primitive root for any integer of the form 17^k .
5. (a) Let p be an odd prime and $\gcd(a, p) = 1$. Then prove that a is a quadratic residue of p if and only

$$\text{if } a^{\frac{p-1}{2}} \equiv 1 \pmod{p}.$$

- (b) Find the value of the Legendre symbol $\left(\frac{-72}{131}\right)$.

- (c) Encrypt the message NUMBER THEORY IS EASY using the linear cipher

$$C \equiv 5P + 11 \pmod{26}.$$

- (d) The ciphertext ALXWU VADCOJO has been enciphered with the cipher

$$C_1 \equiv 4P_1 + 11P_2 \pmod{26}$$

$$C_2 \equiv 3P_1 + 8P_2 \pmod{26}$$

Derive the plaintext.

[This question paper contains 8 printed pages.]

Your Roll No.

Sr. No. of Question Paper : 1358

J

Unique Paper Code : 32357614

Name of the Paper : MATHEMATICAL
FINANCE

Name of the Course : B.Sc. (H) Mathematics
(CBCS-LOCF)

Semester : VI – DSE

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All **six** questions are compulsory, attempt any **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of calculator/scientific calculator and normal distribution table is allowed.

P.T.O.

1. (a) A bank quotes an interest rate of 14% per annum with quarterly compounding. What is the equivalent rate with
 - (i) continuous compounding and
 - (ii) annual compounding?
- (b) Explain the following type of interest rates :
 - (i) Treasury Rates
 - (ii) LIBOR
 - (iii) Repo Rates
- (c) Portfolio A consists of 1-year zero coupon with a face value of ₹2000 and a 10-year zero coupon bond with face value of ₹6000. Portfolio B consists of a 5.95-year zero coupon bond with face value of ₹5000. The current yield on all bonds is 10% per annum.
 - (i) Show that both portfolios have the same duration.

- (ii) What are the percentage changes in the values of the two portfolios for a 5% per annum increase in yields?

(You can use the exponential values: $e^x = 0.905$, 0.368 , 0.552 , 0.861 , 0.223 and 0.409 for $x = -0.1$, -1.0 , -0.595 , -0.15 , -1.5 and -0.893 respectively)

2. (a) Explain Hedging. How is the risk managed when Hedging is done using?

(i) Forward Contracts

(ii) Options

- (b) (i) What is the difference between the over-the counter market and the exchange-traded market?

- (ii) An investor enters into a short forward contract to sell 100,000 British pounds for US dollars at an exchange rate of 1.9000 US dollars per pound. How much does the investor gain or lose if the exchange rate at the end of the contract is (a) 1.8900 and (b) 1.9200?

- (c) Write a short note on European call options. Explain the payoffs in different types of call option positions with the help of diagrams.
3. (a) Name the six factors that affect stock option prices. Explain any three of them.
- (b) Derive the put-call parity for European options on a non-dividend-paying stock. Use put-call parity to derive the relationship between the vega of a European call and the vega of a European put on a non-dividend-paying stock.
- (c) What are the lower and upper bounds for European calls on a non-dividend-paying stock? Calculate the lower bound for the price of a 4-month call option on a non-dividend-paying stock when the stock price is \$28, the strike price is \$25, and the risk-free interest rate is 8% per annum?

4. (a) Consider the standard one-period binomial model where the stock price goes from S_0 to S_0u or S_0d with $d < 1 < u$, and consider an option which pays f_u or f_d in each case, and assume that the interest rate is r and time to maturity is T .

Derive the formula for the no-arbitrage price of the option.

- (b) A stock price is currently ₹50. It is known that at the end of six months it will be either ₹45 or ₹55. The risk-free interest rate is 10% per annum with continuous compounding. What is the value of a six-month European call option with a strike price of ₹50? (You can use exponential value: $e^{-0.05} = 0.95123$).
- (c) A stock price is currently ₹100. Over each of the next two three-month periods it is expected to go up by 8% or down by 7%. The risk-free interest rate is 5% per annum with continuous compounding. What is the value of a six-month American put option with a strike price of ₹102? (You can use exponential value: $e^{-0.0125} = 0.9876$).

5. (a) Given that in a risk-neutral world,

$$\ln S_T \sim \phi \left[\ln S_0 + \left(r - \frac{\sigma^2}{2} \right) T, \sigma^2 T \right], \text{ where } S_T \text{ is the}$$

stock price at a future time T , S_0 is the current stock price, r is the risk-free rate, σ is the volatility and $\phi(m, v)$ denotes a normal distribution with mean m and variance v . For the given strike price K , find $P(K > S_T)$, the probability that a European put option be exercised in a risk-neutral world.

- (b) Calculate the price of a five-month European call option on a non-dividend-paying stock with a strike price of ₹60 when the current stock price is ₹62, the risk-free interest rate is 10% per annum, and the volatility is 20% per annum.

$$(\ln(62/60) = 0.0328)$$

- (c) Show that the Black-Scholes-Merton formulas for call and put options satisfy the put-call parity.

6. (a) Discuss the delta of a European call option and calculate the delta of an at-the-money 6-month European call option on a non-dividend-paying stock when the risk-free interest rate is 8% per annum and the stock price volatility is 30% per annum.

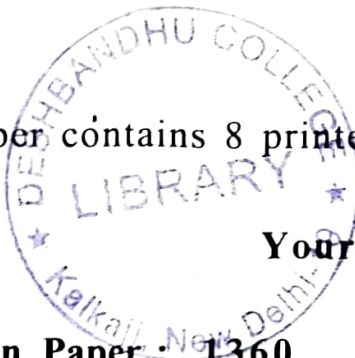
(b) Companies X and Y have been offered the following rates per annum on a \$5 million 10-year investment :

	Fix Rate	Floating Rate
Company X	8.0%	LIBOR
Company Y	8.8%	LIBOR

Company X requires a fixed-rate investment; company Y requires a floating-rate investment. Design a swap that will net a bank, acting as intermediary, 0.2% per annum and will appear equally attractive to X and Y.

- (c) Find the payoff from a bear spread created using put options. Also draw the profit diagram corresponding to this trading strategy.

[This question paper contains 8 printed pages.]



Your Roll No.

27/5/25

Sr. No. of Question Paper : 1360

J

Unique Paper Code : 32357616

Name of the Paper : DSE-4 Linear Programming and Applications

Name of the Course : CBCS (LOCF) – B.Sc. (H) Mathematics

Semester : VI

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions carry equal marks.

1. (a) Define a convex set. Prove that the set S defined as;

$$S = \{(x_1, x_2): x_1^2 + x_2^2 \leq 1\} \text{ is a convex set.}$$

P.T.O.

- (b) Sketch the feasible region of the given linear programming problem and hence find its optimal solution

$$\begin{aligned}\text{Max } Z &= 3x_1 + x_2 \\ \text{s.t. } & -x_1 + 2x_2 \leq 0 \\ & x_2 \leq 4 \\ & x_1, x_2 \geq 0.\end{aligned}$$

- (c) Reduce $2a_1 + 3a_2 + a_3 = b$ to a Basic Feasible Solution, if

$$a_1 = [2, 3], a_2 = [1, 1], a_3 = [4, 5] \text{ and } b = [11, 14].$$

Name the basic and non-basic variables in the reduced Basic Feasible Solution.

2. (a) Apply Two-phase method to find the solution of the following linear programming Problem :

$$\begin{aligned}\text{Min } Z &= 5x_1 + 11x_2 \\ \text{s.t. } & 2x_1 + x_2 \leq 4 \\ & 3x_1 + 4x_2 \geq 24 \\ & 2x_1 - 3x_2 \geq 6 \\ & x_1, x_2 \geq 0.\end{aligned}$$

- (b) Solve the following linear programming problem by Simplex method :

$$\begin{aligned}\text{Max } Z &= 3x_1 + 2x_2 \\ \text{s.t,} \quad x_1 &\leq 4 \\ x_2 &\leq 6 \\ x_1 + x_2 &\leq 5 \\ x_1, x_2 &\geq 0.\end{aligned}$$

- (c) Using Chame's M-technique to solve the following linear programming problem:

$$\begin{aligned}\text{Max } Z &= x_1 + 2x_2 + 3x_3 \\ \text{s.t,} \quad x_1 + x_2 + x_3 &= 5 \\ 2x_2 + x_3 &\geq 2 \\ 2x_1 + x_3 &\leq 10 \\ x_1, x_2, x_3 &\geq 0.\end{aligned}$$

3. (a) Find the dual of the following linear programming problem:

$$\begin{aligned}\text{Max } Z &= 5x_1 + 6x_2 \\ \text{s.t,} \quad x_1 + 2x_2 &= 5 \\ -x_1 + 5x_2 &\geq 3 \\ 4x_1 + 7x_2 &\leq 8 \\ x_1 \text{ unrestricted, } x_2 &\geq 0.\end{aligned}$$

- (b) Solve graphically the dual of given primal linear programming problem, then use complementary slackness condition to determine the solution of the primal problem.

$$\begin{aligned}
 \text{Min } Z &= 2x_1 + x_2 + 3x_3 + 6x_4 \\
 \text{s.t.} \quad &x_1 + x_2 + 3x_3 + 2x_4 \geq 3 \\
 &2x_1 + x_2 + x_3 + 3x_4 \geq 2 \\
 &x_1, x_2, x_3, x_4 \geq 0.
 \end{aligned}$$

- (c) Let x_0 be the feasible solution to the primal problem.

$$\begin{aligned}
 \text{Max } Z &= CX \\
 \text{s.t.} \quad &AX \leq b \\
 &X \geq 0.
 \end{aligned}$$

Where, $X^T, C \in \mathbf{R}^n$, $b^T \in \mathbf{R}^m$ and $A = m \times n$ matrix. If w_0 be a feasible solution to the dual of the primal problem.

$$\begin{aligned}
 \text{Min } Z^* &= b^T w \\
 \text{s.t.} \quad &A^T w \geq C^T \\
 &w \geq 0.
 \end{aligned}$$

Where $w^T \in \mathbf{R}^m$. Then prove that $Cx_0 \leq b^T w_0$.

4. (a) A company has three production houses and supplying the product to five warehouses. The cost of production varies from production houses to production houses and cost of transportation from production houses to warehouses also varies. Each Production houses has a certain production capacity and each warehouses have certain amount of requirement. The cost of transportation per unit is given below.

	Warehouse					
Production House	I	II	III	IV	V	Supply
A	6	4	4	7	5	100
B	5	6	7	4	8	125
C	3	4	6	3	4	175
Demand	60	80	85	105	70	

Determine the optimum quantity to be supplied from each production houses to different warehouses at minimum cost.

- (b) A company wish to hire five employees for five positions claim that they have the required knowledge for each position. Each position can be assigned to one and only one employee. The cost of assignment (per lakh CTC) of position to each employee is given below in the table. Allocate the positions to appropriate employee for optimal assignment.

Positions Employee	A	B	C	D	E
I	3	8	2	10	3
II	8	7	2	9	7
III	6	4	2	7	5
IV	8	4	2	3	5
V	9	10	6	9	10

- (c) An Oil company has three refineries and four depots. The transportation cost per ton, capacity and requirements are given below in the table.

	Depots				
Factories	D ₁	D ₂	D ₃	D ₄	Capacity (tons)
R ₁	5	7	13	10	700
R ₂	8	6	14	13	400
R ₃	12	10	9	11	800
Requirements	200	600	700	400	

Determine the optimum distribution for this company to minimize the shipping cost using North-West Comer Rule, Least Cost Method and Vogel's Approximation Method. Compare the solutions (in terms of costs).

5. (a) Define saddle point. Solve the game whose payoff matrix is given below.

	Player B			
Player A	3	2	4	0
	3	4	2	4
	4	2	4	0
	0	4	0	8

- (b) Find the optimal strategies for both player A and B and value of the game whose payoff matrix is given below.

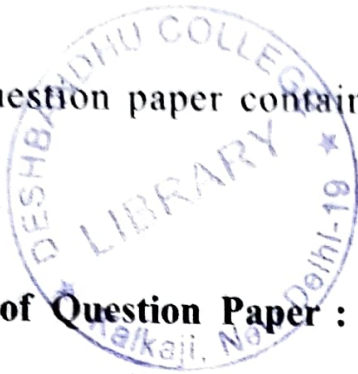
	B ₁	B ₂
A ₁	1	-3
A ₂	3	5
A ₃	-1	6
A ₄	4	1
A ₅	2	2
A ₆	-5	0

- (c) Solve the following system of equations by using Simplex method :

$$x_1 + x_2 = 1$$

$$2x_1 + x_2 = 3.$$

[This question paper contains 4 printed pages.]



15/16

May 2025

Your Roll No.....

Sr. No. of Question Paper : 5514

J

Unique Paper Code : 2352013601

Name of the Paper : Advanced Group Theory

Name of the Course : B.Sc. (H) Mathematics

Semester : VI – DSC-16

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
 2. All questions are compulsory and are of 15 marks each.
 3. Attempt any five parts from Question 1. Each part is of 3 marks.
 4. Attempt any two parts from each of the Questions 2 to 6. Each part is of 7.5 marks.
-
1. (i) Prove that the left regular action of a group G on itself is a faithful action.
(ii) Prove that the kernel of an action of a group G on a set A is a normal subgroup of G .
(iii) Let σ be an m -cycle in the symmetric group S_n . Find $|C_{S_n}(\sigma)|$, the order of centralizer of σ .

P.T.O.

- (iv) Is a group of order 210 simple? Justify.
 - (v) Define commutator subgroup G' of a group G . Prove that G' is normal in G .
 - (vi) Prove that the center of a group is a characteristic subgroup.
 - (vii) Define a solvable group. Prove that every abelian group is solvable.
2. (a) For a group G , consider the mapping $G \times G \rightarrow G$ given by $g.a = gag^{-1}$. Prove that this defines a group action of G on itself. Also, find the kernel of this action and the stabilizer G_x of an element $x \in G$.
- (b) Let $G = D_8$ be the dihedral group of order 8 and let $A = \{1, r, r^2, r^3\}$, where r denotes a clockwise rotation of the square by $\frac{\pi}{2}$ radians. Show that the normalizer $N_{D_8}(A) = D_8$ and the centralizer $C_{D_8}(A) = A$.
- (c) Let p be a prime number and let G be a group of order p^α for some $\alpha \geq 1$. Prove that G has a nontrivial center $Z(G)$. Deduce that every group of order p^2 is abelian.
3. (a) Let the symmetric group S_n act on the set $A = \{1, \dots, n\}$ by $\alpha.a = \alpha(a)$, $\forall \alpha \in S_n, a \in A$. Prove that this action is transitive.

(b) Let the dihedral group D_8 act via its natural action on the set $A = \{1,2,3,4\}$ consisting of four vertices of a square. Label these vertices 1,2,3,4 in a clockwise direction. Let r be the rotation of the square clockwise by $\frac{\pi}{2}$ radians and s be the reflection in the line which passes through vertices 1 and 3. Find the stabilizer of all the four vertices of square.

(c) Let G be a finite group and let $g_1, g_2, \dots, g_r$ be representatives of the distinct conjugacy classes of G not contained in the centre $Z(G)$ of G . Then prove the class equation:

$|G| = |Z(G)| + \sum_{i=1}^r |G : C_G(g_i)|$, where $C_G(g_i)$ is the centralizer of the element g_i in G . Verify the class equation for the symmetric group S_3 .

4. (a) Let G be a finite group and p be a prime dividing the order of G . Let P be a Sylow p -subgroup of G , and let n_p denote the number of Sylow p -subgroups of G . Assume that G acts transitively on the set of its Sylow p -subgroups by conjugation.

(i) Describe the orbit of P under this action.

(ii) Show that $n_p = |G : N_G(P)|$, where $N_G(P)$ denotes the normalizer of P in G .

- (b) Let G be a group of order 12. Prove that either G has a normal Sylow 3-subgroup or G is isomorphic to alternating group A_4 .
- (c) State Sylow's first theorem. Let G be a group of order 77. Prove that G has a normal Sylow 7-subgroup and a normal Sylow 11-subgroup. Further, prove that G is cyclic.
5. (a) State and prove the Index theorem. Use this theorem to show that there is no simple group of order 216.
- (b) Let G be a finite group, and let p be the smallest prime dividing the order of G . Prove that if G has a subgroup of index p , then it must be a normal subgroup of G . Is it always true that G has a subgroup of index p ? Justify.
- (c) Prove that no simple group has order pqr , where p , q and r are distinct primes.
6. (a) Prove that a group G is solvable if and only if $G^{(n)} = \{e\}$ for some positive integer n , where $G^{(n)}$ is the n^{th} derived subgroup of G .
- (b) Find a composition series for the symmetric group S_4 .
- (c) Prove that any finite p -group is nilpotent.

[This question paper contains 8 printed pages.]

Your Roll No.

22/5/258

Sr. No. of Question Paper : 5554

J

Unique Paper Code : 2352013602

Name of the Paper : Advanced Linear Algebra

Name of the Course : Bachelor of Science
(Honours Course)
Mathematics

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory.
3. Attempt any TWO parts from each Question.

P.T.O.

1. (a) Let T be a linear operator on $\mathbb{R}^2$ defined as

$$T \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2a + b \\ a - 3b \end{pmatrix}.$$

For the ordered basis $\beta = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$ and

$\beta' = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$, of $\mathbb{R}^2$, find the change of

coordinate matrix Q that changes β' -coordinates into β -coordinates. Also, verify that

$$[T]_{\beta'} = Q^{-1} [T]_{\beta} Q. \quad (2.5+5)$$

(b) Let $V = P_1(\mathbb{R})$ and for $p(x) \in V$, let $f_1, f_2 \in V^*$ be defined as

$$f_1(p(x)) = \int_0^1 p(t) dt$$

and $f_2(p(x)) = \int_0^2 p(t) dt.$

Prove that $\{f_1, f_2\}$ is a basis for V^* and find a basis of V for which it is the dual basis.

(3+4.5)

- (c) Let V be finite dimensional vector space. Define the annihilator S^0 of a subset S of V and prove that S^0 is a subspace of V^* . If W_1 and W_2 are subspaces of V , prove that

$$(W_1 + W_2)^0 = W_1^0 \cap W_2^0. \quad (3.5+4)$$

2. (a) Let $A = \begin{pmatrix} i & 1 \\ 2 & -i \end{pmatrix} \in M_{2 \times 2}(C)$. Determine all eigen

values of A and for each eigen value λ of A , find the set of eigen vectors corresponding to λ . Also, find a basis for C^2 consisting of eigenvectors of A .

(2.5+5)

P.T.O.

(b) Let T be a linear operator on $P_2(\mathbb{R})$ defined as

$$T(f(x)) = f(0) + f(1)(x + x^2).$$

Test the linear operator T for diagonalizability. If T is diagonalizable, then find a basis β for V such that $[T]_\beta$ is a diagonal matrix. (2.5+5)

(c) Let T be a diagonalizable linear operator on a finite dimensional vector space V and $\lambda_1, \lambda_2, \dots, \lambda_k$ be the distinct eigen values of T . Prove that

$$V = E_{\lambda_1} \oplus E_{\lambda_2} \oplus \dots \oplus E_{\lambda_k}, \text{ where } E_{\lambda_i} \text{ is the eigen space of } \lambda_i, \text{ for all } i. \quad (7.5)$$

3. (a) Let T be a linear operator on the vector space $V = \mathbb{R}^4$ defined as

$$T(a, b, c, d) = (a + b, b - c, a + c, a + d)$$

Find an ordered basis of the T -cyclic subspace W of V generated by $z = e_1$. Also, find the characteristic polynomial of T_W . (3+4.5)

(b) Let T be a linear operator defined on a finite dimensional vector space V . Prove that the characteristic polynomial and the minimal polynomial of T have the same zeros. (7.5)

(c) Let T be a linear operator on $V = M_{n \times n}(\mathbb{R})$ defined as $T(A) = A^t$. Find the minimal polynomial of T . Hence show that T is diagonalizable. (7.5)

4. (a) Let V be an inner product space, prove that the following inequality holds

$$|\langle x, y \rangle| \leq \|x\| \|y\|, \text{ for all } x, y \in V.$$

Also, verify that the inequality holds for $x = (1, 2i, 1 + i)$, $y = (5 + i, 1, 2)$ in $\mathbb{C}^3$. (5+2.5)

(b) Let V be an inner product space and let S be an orthogonal subset of V consisting of nonzero vectors. Prove that S is linearly independent.

(7.5)

- (c) Let $W = \text{span} (\{(i, 0, 1)\})$ in C^3 . Find the orthonormal bases for W and $W^\perp$. (7.5)

5. (a) Let $V = P(R)$ with the inner product defined as

$$\langle f(x), g(x) \rangle = \int_0^1 f(t) g(t) dt, \quad \forall f(x), g(x) \in V.$$

Find the orthogonal projection of the vector $h(x) = 4 + 3x - 2x^2$ on the subspace $W = P_1(R)$. (7.5)

- (b) (i) Let V be a finite dimensional inner product space and β be an orthonormal basis for V . If T is a linear operator on V , show that

$$[T^*]_\beta = ([T]_\beta)^*.$$

- (ii) For the inner product space $V = C^2$ and linear operator

$$T(z_1, z_2) = (2z_1 + iz_2, (1 - i)z_1),$$

evaluate T^* at $z = (3 - i, 1 + 2i)$. (4.5+3)

- (c) Find the best fit linear function for the data $\{(-3,9), (-2,6), (0,2), (1,1)\}$ using the least squares approximation. Also, compute the error E .

(5+2.5)

6. (a) Let T be a normal operator defined on a finite dimensional real inner product space V whose characteristic polynomial splits. Prove that V has an orthonormal basis of eigen vectors of T . Hence prove that T is self-adjoint.

(5+2.5)

- (b) For the following matrix A , find an orthogonal matrix P and a diagonal matrix D such that $P^t A P = D$.

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}. \quad (7.5)$$

- (c) (i) State the Spectral theorem.

- (ii) Let T be a linear operator on a finite dimensional complex inner product space V . Show that T is unitary if and only if T is normal and $|\lambda| = 1$ for every eigen value λ of T . (2+5.5)

This question paper contains 4 printed pages|

Roll No.

(27/5/25) *W*

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S. No. of Question Paper : 5574

Unique Paper Code : 2352013603

Name of the Paper : Complex Analysis

Name of the Course : B.Sc. (Hons.) Mathematics (NEP-UGCF 2022)

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory and carry equal marks.

Each question contains three parts (a), (b) and (c), attempt only two parts.

1. (a) (i) Sketch the set $S = \left\{ z \in \mathbf{C} : |z| \leq 1, -\frac{\pi}{2} \leq \arg z \leq \frac{\pi}{2} \right\}$. 3

(ii) Under the map $f(z) = z^2$, draw the image of the set : 4.5

$$S = \left\{ z \in \mathbf{C} : |z| = 1, -\frac{\pi}{4} \leq \arg z \leq \frac{\pi}{4} \right\}.$$

(b) Under the map $f(z) = e^z$ draw the image of the rectangular region

$$R = \left\{ z = x + iy \in \mathbf{C} : 1 \leq x \leq 2, -\frac{\pi}{4} \leq y \leq \frac{\pi}{2} \right\}. 7.5$$

P.T.O.

(c) Does $\lim_{z \rightarrow 0} \left(\frac{z}{\bar{z}} \right)^2$ exist ? Justify your answer. 7.5

2. (a) Show that the function $f(z) = \bar{z}$ is nowhere differentiable on $\mathbb{C}$. 7.5

(b) (i) Giving proper justification, write two differences between the functions : 3.5

$$f(x) = e^x, x \in \mathbf{R} \text{ and } f(z) = e^z, z \in \mathbf{C}$$

(ii) Solve the equation $e^z = -2$. 4

(c) Find out the values of $\sin^{-1} 2$. 7.5

3. (a) (i) Let a function f be integrable along a contour C . Prove that : 3.5

$$\int_{-C} f(z) dz = - \int_C f(z) dz.$$

(ii) Define a simple arc. Give an example to explain that a same set of points may represent different arcs. 4

(b) Let C be any simple closed contour, described in the positive sense in the z plane, then compute the value of the integral :

$$\int_C \frac{s^3 + 2s}{(s - z_0)^3} ds,$$

(i) when z_0 is inside C , and

(ii) when z_0 is outside C . 7.5

(c) (i) Let $w(t)$ be a piecewise continuous complex valued function defined on an interval $a \leq t \leq b$, then show that : 3.5

$$\left| \int_a^b w(t) dt \right| \leq \int_a^b |w(t)| dt.$$

- (ii) Let C be the arc of the circle $|z| = 2$ such that $0 \leq \arg z \leq \frac{\pi}{2}$.
Show that :

$$\left| \int_C \frac{z+4}{z^3-1} dz \right| \leq \frac{6\pi}{7}.$$

4. (a) State and prove Cauchy integral formula. 7.5
- (b) Let f be an entire function such that $|f(z)| \leq A|z|$ for all z , where A is a fixed positive number. Show that $f(z) = bz$, where b is a complex constant. 7.5
- (c) (i) Evaluate the following integral along the line segment, say L , from 0 to $\pi + 2i$: 3.5

$$\int_L \cos(z/2) dz.$$

- (ii) Evaluate the following integral :

$$\int_C \frac{z dz}{(9-z^2)(z+i)},$$

where C is the positively oriented circle $|z| = 2$.

4

5. (a) State Laurent's series theorem. Give two series expansions in powers of z for the function $f(z) = \frac{z+1}{z-1}$ specifying the regions in which those expansions are valid. 7.5
- (b) If a power series $\sum_{n=0}^{\infty} a_n (z-z_0)^n$ converges when $z = z_1$ ($z_1 \neq z_0$), then prove that it is absolutely convergent at each point z in the open disk $|z-z_0| < R_1$ where $R_1 = |z_1 - z_0|$. 7.5

- (c) Define residue. Evaluate the residue of the function $f(z) = \frac{e^z - 1}{z^4}$ and use

it to find the integral $\int_C \frac{e^z - 1}{z^4} dz$, where C is the positively oriented unit circle $|z| = 1$. 7.5

6. (a) Define an isolated singular point. For the below given functions, find the isolated singular point and determine whether it is a removable singular point, an essential singular point, or a pole : 7.5

(i) $\frac{\sin z}{z}$ and

(ii) $\frac{e^{2z}}{(z-1)^2}$.

- (b) Let z_0 be an isolated singular point of a function f . Then show that the following two statements are equivalent :

(i) z_0 is a pole of order m ($m = 1, 2, \dots$) of f ;

(ii) $f(z)$ can be written in the form $f(z) = \frac{\varphi(z)}{(z-z_0)^m}$, where $\varphi(z)$ is analytic

and non-zero at z_0 .

Further, establish that

$$\operatorname{Res}_{z=z_0} f(z) = \frac{\varphi^{(m-1)}(z_0)}{(m-1)!} \text{ when } m = 1, 2, 3, \dots$$

where $\varphi^{(0)} \equiv \varphi$.

7.5

- (c) Using residues, evaluate the following integral :

7.5

$$\int_0^{2\pi} \frac{d\theta}{5 + 4 \sin \theta} d\theta.$$

This question paper contains 8 printed pages]

02/6/25 *

Roll No.

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S. No. of Question Paper : 8082

Unique Paper Code : 2353010013

Name of the Paper : Mathematical Finance

Type of the Paper : DSE

Name of the Course : B.Sc. (Hons.) Mathematics

Semester : VI

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any two parts from each question.

All questions are compulsory and carry equal marks.

Use of scientific calculator, basic calculator and

normal distribution tables all are allowed.

1. Attempt any two parts :

- (a) Define the term derivative contract with an example. What is the difference between a long position and a short position in a forward contract ? Suppose that on May 01, 2025, the treasurer of a U.S. Corporation knows that the Corporation will pay £ 10 million in 6

P.T.O.

months (i.e., on November 01, 2025). The bid and ask 6-month forward quotes or the exchange rate between USD and GBP (USD per GBP) on May 01, 2025, are 1.3091 and 1.3099, respectively. How can the corporation hedge its position against exchange rate moves ?

- (b) What are the *three* types of traders ? Explain the arbitrage opportunity when the price of a dually listed technology company stock is \$40 (USD) on the NASDAQ and ₹ 3,500 (INR) on the National Stock Exchange of India (NSE). Assume that the exchange rate is ₹ 83 per U.S. dollar. Explain what is likely to happen to prices as traders take advantage of this opportunity.
- (c) Let R_c denote rates of interest with continuous compounding and R_m denote equivalent rates with compounding m times per annum. Derive the relation between R_2 and R_4 . An Indian investor is comparing two fixed-income investment options. One investment offers a semi-annual interest rate of 6% per annum, while another offers a quarterly interest rate. What is the equivalent quarterly interest rate that would yield the same return as the 6% semi-annual rate ? Express your answer as a percentage rounded to two decimal places.

2. Attempt any *two* parts :

- (a) Define bond price, bond yield, and par yield. Suppose that the 6-month, 12-month, 18-month, and 24-month zero rates are 4%, 5.5%, 6.2% and

6.8% per annum, respectively (with continuous compounding). What is the bond price and 2-year par yield ? (You can use the exponential values : $e^{-0.02} = 0.9802$, $e^{-0.055} = 0.9465$, $e^{-0.0930} = 0.9112$, $e^{-0.136} = 0.8728$)

- (b) If R_1 and R_2 are the zero rates for maturities T_1 and T_2 , respectively and R_F is the forward interest rate for the period of time between T_1 and T_2 . Write an expression for R_F . Suppose that the risk-free zero-coupon interest rates for 1 year, 2 years and 3 years are 4.0%, 4.4% and 4.8%, respectively. Calculate forward interest rates for the second and third years. Explain the following statement : "When the zero curve is upward-sloping, the zero rate for a particular maturity is greater than the par yield for that maturity. When the zero curve is downward-sloping, the reverse is true."
- (c) Define the duration of a bond. What is the duration of a zero-coupon bond ? Consider a 5-year bond issued by an Indian company with a face value of ₹ 1,000 and an annual coupon rate of 8%, paid yearly. If the yield on the bond is 9% per annum with continuous compounding, what is the duration of the bond in years ? (You can use the exponential values : $e^{-0.09} = 0.9139$, $e^{-0.18} = 0.8353$, $e^{-0.27} = 0.7634$, $e^{-0.36} = 0.6977$, $e^{-0.45} = 0.6376$)

3. Attempt any two parts :

- (a) Distinguish between a European option and an American option. Additionally, explain the concepts of a long position in a call option and a short position in a put option. Illustrate each with their respective payoff functions and payoff diagrams.
- (b) Show that the lower bound for the price of a European call option on a non-dividend-paying stock when the stock price is S_0 , the strike price is K , the risk-free interest rate is $r\%$, and the time to expiration of option is T , is given by $S_0 - Ke^{-rT}$. Also, what is the lower bound for the price of a one-month European put option on a non-dividend paying stock when the stock price is \$12, the strike price is \$15, and the risk-free interest rate is 6% per annum ? (You can use the exponential value : $e^{-0.0049} = 0.995$).
- (c) Derive the put-call parity relation for a European stock option on a non-dividend paying stock. The price of a non-dividend paying stock is \$19 and the price of a three-month European call option on the stock with a strike price of \$20 is \$1. The risk-free rate is 4% per annum. What is the price of a three-month European put option with a strike price of \$20 ? (You can use the exponential value : $e^{-0.01} = 0.990$).

4. Attempt any two parts :

- (a) Explain how a bull spread can be created using two European call options on a stock ? Write the payoff table along with profit diagram corresponding to this trading strategy. A call option with a strike price of \$50 costs \$2. A put option with a strike price of \$45 costs \$3. Explain how a strangle can be created from these two options. What is the pattern of profits from the strangle ?
- (b) Define briefly the no-arbitrage valuation approach to valuing a European option. A stock price is currently \$40. It is known that at the end of one month it will be either \$42 or \$38. The risk-free interest rate is 8% per annum with continuous compounding. What is the value of a one-month European call option with a strike price of \$39 ? (You can use the exponential value : $e^{-0.0066} = 0.993$).
- (c) Define briefly what is meant by risk neutral world in the context of valuing a European option using a binomial tree. A stock price is currently \$100. Over each of the next two six-month periods it is expected to go up by 10% or down by 10%. The risk-free interest rate is 8% per annum with continuous compounding. What is the value of a one-year European call option with a strike price of \$100 ? (You can use the exponential values : $e^{0.04} = 1.040$, $e^{-0.08} = 0.923$).

5. Attempt any two parts :

(a) Stock price in the Black-Scholes model satisfies :

$$\ln S_T \sim \phi \left[\ln S_0 + \left(\mu - \frac{\sigma^2}{2} \right) T, \sigma^2 T \right]$$

where $\phi(m, v)$ denotes a normal distribution with mean m and variance v . Find $P(S_T < K)$.

(b) Stock price in the Black-Scholes model satisfies :

$$\ln S_T \sim \phi \left[\ln S_0 + \left(\mu - \frac{\sigma^2}{2} \right) T, \sigma^2 T \right]$$

where $\phi(m, v)$ denotes a normal distribution with mean m and variance v . Find $E[S_T]$ and $E[S_T^2]$.

(c) Write a short note on stop-loss strategy. Calculate the delta of an at-the-money six-month European call option on a non-dividend-paying stock when the risk-free interest rate is 10% per annum and the stock price volatility is 25% per annum.

6. Attempt any two parts :

(a) Let V be a lognormal random variable with ω being the standard deviation of $\ln V$. Prove that $E[\max(K - V, 0)] = KN(-d_2) - E(V)N(-d_1)$, where

$$d_1 = \frac{\ln \left[\frac{E(V)}{K} \right] + \frac{\omega^2}{2}}{\omega},$$

$$d_2 = \frac{\ln \left[\frac{E(V)}{K} \right] - \frac{\omega^2}{2}}{\omega},$$

and E denotes the expected value. Use this result to derive the Black-Scholes formula for the price of a European put option on a non-dividend paying stock.

(b) Define delta and gamma of a portfolio. Suppose a financial institution has the following three positions on a stock :

- (i) A long position in 100,000 call options with strike price \$55 and an expiration date in 3 months. The delta of each option is 0.533.
- (ii) A short position in 200,000 call options with strike price \$56 and an expiration date in 5 months. The delta of each option is 0.468.
- (iii) A short position in 50,000 put options with strike price \$56 and an expiration date in 2 months. The delta of each option is -0.508.

Calculate delta of the whole portfolio and how portfolio can be made delta-neutral. Suppose that gamma of this delta-neutral portfolio of options on an asset is -10,000. If there is a change of +3 or -3 in the price of an asset over a short period of time, what is the change in the value of the portfolio ?

- (c) In Black-Scholes model, the stock prices follow the Geometric Brownian motion given by :

$$dS = \mu S dt + \sigma S dz$$

Using Itô's Lemma on $\ln S$, show that :

$$\ln S_T \sim \phi \left[\ln S_0 + \left(\mu - \frac{\sigma^2}{2} \right) T, \sigma^2 T \right]$$